EQUAL + LIBRIUM

# OUR DEFINITION OF EQUILIBRIUM (THE EQUILIBRIUM CONDITIONS)

## AND THE IMPLICIT ROLE OF TIME

THE MARKET DEMAND FUNCTION $\overset{\Delta}{X}(\cdot)$ TELLS US, FOR EACH PRICE-LIST $p \in \mathbb{R}^{\ell}_{+}$, WHAT $\overset{\Delta}{X} \in \mathbb{R}^{\ell}$ WILL BE.
SUPPOSE WE ALSO KNEW WHAT $\Delta p \in \mathbb{R}^{\ell}$ WOULD BE FOR ANY EXCESS DEMAND VECTOR $\overset{\Delta}{X} \in \mathbb{R}^{\ell}$. (DISCRETE TIME)

THEN THE "TIME LINE" WOULD LOOK LIKE THIS:

$$\begin{tikzcd} p \arrow[r, mapsto, "\overset{\Delta}{X}(\cdot)"] \arrow[rrr, bend right, "f(\cdot)"'] & \overset{\Delta}{X} \arrow[r, mapsto, "?"] & \Delta p \arrow[r, mapsto, "p+\Delta p"] & p \arrow[r, mapsto] & \overset{\Delta}{X} \arrow[r, mapsto] & \Delta p \arrow[r] & \cdots \end{tikzcd}$$

CALL $p$ THE "STATE" OF THE SYSTEM. FROM $p$ WE CAN DETERMINE $\overset{\Delta}{X}$ AND $x_i$ FOR EACH PERSON $i$.
IF WE KNEW THE ANSWER TO ? WE COULD DETERMINE, FOR ANY STATE $p \in \mathbb{R}^{\ell}$, WHAT THE NEXT STATE, SAY $p'$, WOULD BE. WE COULD WRITE THAT AS A FUNCTION:

$$p' = f(p) \qquad f: S \to S$$

☆ WHAT WOULD BE AN EQUILIBRIUM OF THIS SYSTEM?
<u>ANSWER:</u> A $p \in S$ SUCH THAT $\Delta p = \underline{0}$ — i.e., $p = f(p)$.

UNFORTUNATELY, WE DON'T KNOW THE ANSWER TO (?).
BUT WE GENERALLY KNOW A PARTIAL ANSWER:

$$\Delta p_k \gtreqless 0 \quad \text{AS} \quad \hat{X}_k \gtreqless 0$$

OR AT LEAST $\Delta p_k = 0 \Leftrightarrow \hat{X}_k = 0$.

SO . . . WHILE $\Delta p = \underline{0}$ IS WHAT WE REALLY MEAN BY "EQUILIBRIUM,"

AS A SURROGATE WE USE $\hat{X} = \underline{0}$

i.e., A $p \in \mathbb{R}^\ell_+$ SUCH THAT $\hat{X}(p) = \underline{0}$.

"ALL MARKETS CLEAR"

"NO EXCESS DEMAND OR SUPPLY"

"DEMAND = SUPPLY"